Class XI Session 2024-25
Subject - Mathematics
Sample Question Paper - 5

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. The value of $\sin^2 \frac{5\pi}{12} - \sin^2 \frac{\pi}{12}$ is **[1]**

a) $\sqrt{3}/2$ b) $\frac{1}{2}$

c) 0 d) 1

2. The domain of the function $f(x) = \sqrt{x-1} + \sqrt{6-x}$ **[1]**

a) (-∞, 6) b) [2, 6]

c) [1, 6] d) [-2, 6]

3. If the mean of the squares of first n natural numbers be 11, then n is equal to **[1]**

a) 5 b) $\frac{-13}{2}$

c) 11 d) 13

4. If f(x) = x sinx, then f' $\left(\frac{\pi}{2}\right)$ is equal to **[1]**

a) 1 b) $\frac{1}{2}$

c) -1 d) 0

5. The equation of the line passing through (1, 2) and perpendicular to x + y + 7 = 0 is **[1]**

a) y – x – 1 = 0 b) y – x + 1 = 0

c) y – x + 2 = 0 d) y – x – 2 = 0.

6. Perpendicular distance of the point (3, 4, 5) from the y-axis is, **[1]**

a) $\sqrt{34}$ b) 4

c) $\sqrt{41}$ d) 5

7. If x + iy $= \frac{3+5i}{7-6i}$, then y = [1]

a) $\frac{43}{85}$
b) $\frac{-9}{85}$
c) $\frac{9}{85}$
d) $\frac{53}{85}$

8. A polygon has 44 diagonals. The number of its sides is [1]

a) 8
b) 11
c) 12
d) 10

9. $\lim\limits_{x \to \frac{\pi}{2}} \frac{1-\sin x}{\cos x}$ is equal to [1]

a) 1
b) 0
c) -1
d) does not exit

10. A pendulum swings through an angle of 42° in describing an arc of length 55 cm. The length of the pendulum is [1]

a) 88 cm
b) 60 cm
c) 56 cm
d) 75 cm

11. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n are [1]

a) 7, 4
b) 6, 4
c) 3, 3
d) 6, 3

12. $\sum_{r=0}^{n} 4^r.^n C_r$ is equal to [1]

a) 6^n
b) 5^{-n}
c) 4^n
d) 5^n

13. $\{C_1 + 2C_2 + 3C_3 + ... + nC_n\}$ = ? [1]

a) $(n - 1) \cdot 2^n$
b) $n \cdot 2^n$
c) $(n + 1) \cdot 2^n$
d) $n \cdot 2^{n-1}$

14. Solve: 3x + 5 < x - 7, when x is a real number [1]

a) x > -12
b) x < -12
c) x < -6
d) x > -6

15. Let R be set of points inside a rectangle of sides a and b (a, b > 1) with two sides along the positive direction of x-axis and y-axis. Then [1]

a) R = {(x, y) : 0 $\leq$ x $\leq$ a, 0 $\leq$ y $\leq$ b}
b) R = {(x, y) : 0 $\leq$ x < a, 0 $\leq$ y $\leq$ b}
c) R = {(x, y) : 0 < x < a, 0 < y < b}
d) R = {(x, y) : 0 $\leq$ x $\leq$ a, 0 < y < b}

16. The value of sin 78° - sin 66° - sin 42° + sin 60° is [1]

a) None of these
b) $\frac{1}{2}$
c) -1
d) $\frac{-1}{2}$

17. If f (x) = $\sqrt{1-x^2}$, x $\in$ (0, 1), then f'(x), is equal to [1]

a) $\sqrt{1-x^2}$

b) $\sqrt{x^2-1}$

c) $\frac{1}{\sqrt{1-x^2}}$

d) $\frac{-x}{\sqrt{1-x^2}}$

18. If P(n, r) = C(n, r) then **[1]**

a) r = 0 or 2

b) r = 1 or n

c) r = 0 or 1

d) n = r

19. **Assertion (A):** if A = set of letters in **Alloy** B = set of letters in **LOYAL**, then set A & B are equal sets. **[1]**
Reason (R): If two sets have exactly the same elements, they are called equal sets.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The sum of first 6 terms of the GP 4, 16, 64, ... is equal to 5460. **[1]**
Reason (R): Sum of first n terms of the G.P is given by $S_n = \frac{a(r^n-1)}{r-1}$, where a = first term r = common ratio and |r| > 1.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Write the range of the function f (x) = sin [x], where $\frac{-\pi}{4} \leq x \leq \frac{\pi}{4}$. **[2]**

OR

Find the domain and the range of the real function: f(x) = $\frac{1}{\sqrt{x^2-1}}$

22. Evaluate: $\lim\limits_{x \to 0} \frac{1-\cos 2x}{x^2}$. **[2]**

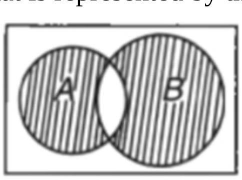
23. Determine the probability p, for event. An odd number appears in a single toss of a fair die. **[2]**

OR

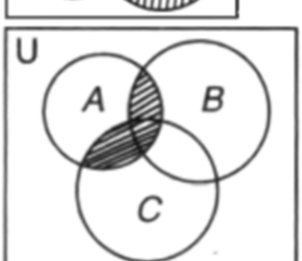
A five digit number is formed by the digits 1, 2, 3, 4, 5 without repetition. Find the probability that the number is divisible by 4.

24. What is represented by the shaded regions in each of the following Venn-diagrams. **[2]**

i.



ii.



25. Find the equations of the lines which cut-off intercepts on the axes whose sum and product are 1 and -6 respectively. **[2]**

Section C

26. It is required to seat 5 men and 3 women in a row so that the women occupy the even places. How many such **[3]**

arrangements are possible?

27. Verify that (0, 7, 10), (-1, 6, 6) and (-4, 9, 6) are the vertices of a right-angled triangle. **[3]**

28. Find a if the coefficient of x^2 and x^3 in the expansion of $(3 + ax)^9$ are equal. **[3]**

OR

Expand $(1 + x + x^2)^3$ using binomial expansion.

29. Evaluate $\lim_{x\to 2}\left(\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}\right)$ **[3]**

OR

Evaluate $\lim_{x\to \pi}\frac{\sin(\pi-x)}{\pi(\pi-x)}$

30. If pth, qth and rth terms of an A.P. and G.P. are both a, b, and c respectively. Show that **[3]**

$a^{b-c} \cdot b^{c-a} \cdot c^{a-b} = 1$

OR

If the sum of an infinite decreasing G.P. is 3 and the sum of the squares of its term is $\frac{9}{2}$, then write its first term and common difference.

31. For any sets A and B show that **[3]**

i. $(A \cap B) \cup (A - B) = A$
ii. $A \cup (B - A) = A \cup B$

Section D

32. From the frequency distribution consisting of 18 observations, the mean and the standard deviation were found to be 7 and 4, respectively. But on comparison with the original data, it was found that a figure 12 was miscopied as 21 in calculations. Calculate the correct mean and standard deviation. **[5]**

33. Find the (i) lengths of major and minor axes, (ii) coordinate of the vertice, (iii) coordinate of the foci, (iv) eccentricity, and (v) length of the latus rectum of ellipse: $16x^2 + 25y^2 = 400$. **[5]**

OR

Show that the equation $x^2 - 2y^2 - 2x + 8y - 1 = 0$ represents a hyperbola. Find the coordinates of the centre, lengths of the axes, eccentricity, latusrectum, coordinates of foci and vertices and equations of directrices of the hyperbola.

34. Solve for x, $\frac{|x+3|+x}{x+2} > 1$ **[5]**

35. Prove that $\cos 12^o + \cos 60^o + \cos 84^o = \cos 24^o + \cos 48^o$ **[5]**

OR

Prove that: $\sin 5x = 5 \sin x - 20 \sin^3 x + 16 \sin^5 x$.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

Representation of a Relation

A relation can be represented algebraically by roster form or by set-builder form and visually it can be represented by an arrow diagram which are given below

i. **Roster form** In this form, we represent the relation by the set of all ordered pairs belongs to R.
ii. **Set-builder form** In this form, we represent the relation R from set A to set B as R = {(a, b): $a \in A$, $b \in B$ and the rule which relate the elements of A and B}.
iii. **Arrow diagram** To represent a relation by an arrow diagram, we draw arrows from first element to second element of all ordered pairs belonging to relation R.

Questions:

i. If n(A) = 3 and B = {2, 3, 4, 6, 7, 8} then find the number of relations from A to B. (1)
ii. If A = {a, b} and B = {2, 3}, then find the number of relations from A to B. (1)
iii. If A = {a, b} and B = {2, 3}, write the relation in set-builder form. (2)

OR

Express of R = {(a, b): 2a + b = 5; a, b $\in$ W} as the set of ordered pairs (in roster form). (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

There are 4 red, 5 blue and 3 green marbles in a basket.

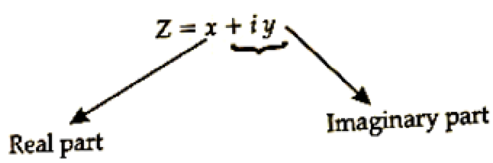
i. If two marbles are picked at randomly, find the probability that both red marbles. (1)
ii. If three marbles are picked at randomly, find the probability that all green marbles. (1)
iii. If two marbles are picked at randomly then find the probability that both are not blue marbles. (2)

OR

If three marbles are picked at randomly, then find the probability that atleast one of them is blue. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

A number of the form Z = x + iy, where x and y are real and i = $\sqrt{-1}$ is called a complex number. Consider the complex number Z_1 = 2 + 3i and Z_2 = 4 - 3i.



i. Find the imaginary part of $Z_1\overline{Z_1}$. (1)
ii. Find the real part of $\frac{z_1}{z_2}$. (1)
iii. Find the imaginary part of Z_1 - Z_2. (2)

OR

Find the real part of Z_1. (2)

Solution

Section A

1. **(a)** $\sqrt{3}/2$

Explanation: $\frac{5\pi}{12} = 75^{\circ}, \frac{\pi}{12} = 15^{\circ}$

$\sin^2 75^o - \sin^2 15^o$

$= \sin^2 75^o - \cos^2 75^o$ $[\sin(90^{\circ} - \theta) = \cos\theta]$

Now, $\sin 75^o = \sin(45^o + 30^o)$

$= \sin 45^o \cos 30^o + \cos 45^o \sin 30^o$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\cos 75^o = \cos(45^o + 30^o)$

$= \cos 45^o \cos 30^o - \sin 45^o \sin 30^o$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

Hence,

$\sin^2 75^o - \cos^2 75^o = \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)^2 - \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2$

$= \frac{3+1+2\sqrt{3}-3-1+2\sqrt{3}}{8}$

$= \frac{4\sqrt{3}}{8}$

$= \frac{\sqrt{3}}{2}$

2.

(c) [1, 6]

Explanation: For f(x) to be real, we must have,

$x - 1 \geqslant 0 \; and \; 6 - x \geqslant 0$

$\Rightarrow \; x \geqslant \varphi \; and \; x - 6 \; \leqslant 6$

$\therefore$ Domain = [1, 6]

3. **(a)** 5

Explanation: Mean = $\frac{\frac{n(n+1)(2n+1)}{6}}{n} = \frac{(n+1)(2n+1)}{6}$

$\Rightarrow$ 11 = $\frac{(n+1)(2n+1)}{6}$

$\Rightarrow$ 66 = (n + 1)(2n + 1)

$\Rightarrow$ $2n^2 + 3n - 65 = 0$

$\Rightarrow$ $2n^2 + 13n - 10n - 65 = 0$

$\Rightarrow$ (2n + 13)(n - 5) = 0

$\Rightarrow$ n = 5, $\frac{-13}{2}$

So, n = 5

4. **(a)** 1

Explanation: f′ (x) = x cosx + sinx

So, $f'\left(\frac{\pi}{2}\right) = \frac{\pi}{2}\cos\frac{\pi}{2} + \sin\frac{\pi}{2} = 1$

5. **(a)** y – x – 1 = 0

Explanation: Suppose the slope of the line be m. Then, its equation passing through (1, 2) is given by

y – 2 = m (x – 1) ... (1)

Again, this line is perpendicular to the given line x + y + 7 = 0 whose slope is – 1

Thus, we have m (– 1) = – 1 or m = 1

Thus, the required equation of the line is obtained by substituting thethe value of m in (1), i.e.,
y – 2 = x – 1 or y – x – 1 = 0

6. **(a)** $\sqrt{34}$

Explanation: Distance of (α, β, γ) from y-axis is given by d = $\sqrt{\alpha^2 + \gamma^2}$

$\therefore$ Distance (d) of (3, 4, 5) from y-axis is

d = $\sqrt{3^2 + 5^2}$ = $\sqrt{9 + 25}$ = $\sqrt{34}$

7.

(d) $\frac{53}{85}$

Explanation: $\frac{53}{85}$

x + iy $= \frac{3+5i}{7-6i}$

$\Rightarrow$ x + iy $= \frac{3+5i}{7-6i} \times \frac{7+6i}{7+6i}$

$\Rightarrow$ x + iy $= \frac{21+53i+30i^2}{49-36i^2}$

$\Rightarrow$ x + iy $= \frac{21-30+53i}{49-36i}$

$\Rightarrow$ x + iy $= \frac{-9}{85} + i\frac{53}{85}$

On comparing both the sides:

$y = \frac{53}{85}$

8.

(b) 11

Explanation: We have an n sided polygon has n vertices. If you join every distinct pair of vertices you will get lines. These lines account for the n sides of the polygon as well as for the diagonals.

So the number of diagonals is given by ${}^nC_2 - n = \frac{n(n-1)}{2} - n = \frac{n(n-3)}{2}$

But number of diagonals = 44

$\Rightarrow$ 44 = $\frac{n(n-3)}{2}$

$\Rightarrow$ 88 = n(n - 3)

$\Rightarrow$ n^2 - 3n - 88 = 0

$\Rightarrow$ (n - 11)(n + 8) = 0

$\Rightarrow$ n = 11, -8

Since n cannot be negative, we get n = 11

9.

(b) 0

Explanation: $\lim_{x \to \frac{\pi}{2}} \frac{1-\sin x}{\cos x} = \lim_{y \to 0} \frac{1-\sin(\frac{\pi}{2}-y)}{\cos(\frac{\pi}{2}-y)} \quad \text{taking } \frac{\pi}{2} - x = y$

$= \lim_{y \to 0} \frac{1-\cos y}{\sin y} = \lim_{y \to 0} \frac{2\sin^2 \frac{y}{2}}{2\sin\frac{y}{2}\cos\frac{y}{2}}$

$= \lim_{y \to 0} \tan\frac{y}{2} = 0$

10.

(d) 75 cm

Explanation: $\theta = 42^\circ = \left(42 \times \frac{\pi}{180}\right)^c = \left(\frac{7\pi}{30}\right)^c$ and l = 55 cm.

$\therefore r = \frac{l}{\theta} = \left(55 \times \frac{30}{7\pi}\right)\text{cm} = \left(55 \times \frac{30}{7} \times \frac{7}{22}\right)\text{cm}$ = 75 cm.

11.

(d) 6, 3

Explanation: Let A and B be two sets having m and n elements respectively. Then,

Number of subsets of A = 2^m, Number of subsets of B = 2^n

It is given that $2^m - 2^n = 56$

So, $2^n(2^{m-n} - 1) = 2^3(2^3 - 1)$

n = 3 and m - n = 3 $\Rightarrow$ n = 3 and m = 6.

12.

(d) 5^n

Explanation: $\sum_{r=0}^{n} 4^r.^n C_r$ = $4^0 \cdot^n C_0 + 4^1 \cdot^n C_1 + 4^2 \cdot^n C_2$ + ... + $4^n \cdot^n C_n$

= $1 + 4.^n C_1 + 4^2.^n C_2$ + ... + $4^n \cdot^n C_n$

= $(1 + 4)^n = 5^n$

13.

(d) $n \cdot 2^{n-1}$

Explanation: $C_1 + 2C_2 + 3C_3 + ... + nC_n = n + 2 \cdot \frac{n(n-1)}{2} + 3 \cdot \frac{n(n-1)(n-2)}{3!}$ + ... + n

= n . [1 + (n - 1) $\frac{(n-1)(n-2)}{2!}$ + ... + 1]

= n.[$^{(n-1)}C_0 + ^{(n-1)}C_1 + ^{(n-1)}C_2 + ... + ^{(n-1)}C_{n-1}$]

= n .$(1 + 1)^{n-1}$ = n.2^{n-1}

14.

(c) x < -6

Explanation: 3x + 5 < x - 7

⇒ 3x + 5 - x < x - 7 - x

⇒ 2x + 5 < -7

⇒ 2x + 5 - 5 < -7 - 5

⇒ 2x < -12

⇒ $\frac{2x}{2}$ < - $\frac{12}{2}$

⇒ x < -6

15.

(c) R = {(x, y) : 0 < x < a, 0 < y < b}

Explanation: We have, R be set of points inside a rectangle of sides a and b

Since, a, b > 1

a and b cannot be equal to 0

Thus,R = {(x, y) : 0 < x < a, 0 < y < b}

16. **(a)** None of these

Explanation: sin 78^o - sin 66^o - sin 42^o + sin 60^o

= sin 78^o - sin 42^o - sin 66^o + sin 60^o

= 2 sin $\left(\frac{78^\circ-42^\circ}{2}\right)$ cos $\left(\frac{78^\circ+42}{2}\right)$ - sin 66^o + sin 60^o [∵ sin A - sin B = 2 sin $\left(\frac{A-B}{2}\right)$ cos $\left(\frac{A+B}{2}\right)$]

= 2 sin 18^o cos 60^o - sin 66^o + sin 60^o

= $2 \times \frac{1}{2}$ sin 18^o - sin 66^o + $\frac{\sqrt{3}}{2}$

= sin 18^o - sin 66^o + $\frac{\sqrt{3}}{2}$

= 0.309 - 0.914 + 0.866

= 0.261

17.

(d) $\frac{-x}{\sqrt{1-x^2}}$

Explanation: f(x) = $\sqrt{1-x^2}$

f'(x) = $\frac{1}{2\sqrt{1-x^2}} - 2x$ = $\frac{-x}{\sqrt{1-x^2}}$

18.

(c) r = 0 or 1

Explanation: Given P(n, r) = C(n, r)

⇒ $\frac{n!}{(n-r)!} = \frac{n!}{r!(n-r)!}$

⇒ 1 = $\frac{1}{r!}$

⇒ r! = 1

⇒ r = 0 or r = 1 [∵ 0! = 1, 1! = 1]

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Assertion: Given GP 4, 16, 64, ...

$\therefore$ a = 4, r = $\frac{16}{4}$ = 4 > 1

$\therefore$ S_6 = $\frac{4\left((4)^6-1\right)}{4-1} = \frac{4(4095)}{3}$ = 5460

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

Section B

21. From the given question , we can write,

f(x) = sin (x)

$-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$

Sin $\left[-\frac{\pi}{4}\right]$ = sin (-1)

= - sin 1

sin 0 = 0

sin $\frac{\pi}{4} = \sin 0$

=0

using properties of greatest integer function

(1) = 1. (0.5) = 0. (0.5) = -1

Hence, R(f) = -(sin 1.0)

OR

Here we have, f(x) = $\frac{1}{\sqrt{x^2-1}}$

we need to find where the function is defined

The condition for the function to be defined

x^2 - 1 > 0

$\Rightarrow$ x^2 > 1

$\Rightarrow$ x > 1

So, the domain of the function is the set of all the real numbers greater than 1

The domain of the function, $D_{\{f(x)\}}$ = (1, ∞)

Now put any value of x within the domain set we get the value of the function always a fraction whose denominator is not equalled to 0

The range of the function, $R_{f(x)}$ = (0, 1).

22. We have to find the value of

$\lim\limits_{x \to 0} \frac{1-\cos 2x}{x^2} = \lim\limits_{x \to 0} \frac{2\sin^2 x}{x^2}$

$= 2\lim\limits_{x \to 0}\left(\frac{\sin x}{x} \times \frac{\sin x}{x}\right) = 2\lim\limits_{x \to 0} \frac{\sin x}{x} \times \lim\limits_{x \to 0} \frac{\sin x}{x}$ = 2 (1) (1) = 2

23. When a fair die is thrown, the possible outcomes are

S = {1, 2, 3, 4, 5, 6}

$\therefore$ Total number of outcomes =$n(S)$= 6 and

Odd numbers in a throw={1, 3, 5}

$\therefore$ Number of favourable outcomes = $n(A)$=3

We know that,

Required probabiltiy $= \frac{n(A)}{n(S)} = \frac{\text{Number of favourable outcome}}{\text{Total number of outcomes}}$ = $\frac{3}{6} = \frac{1}{2}$

OR

We have to find the probability that the number is divisible by 4.

Total number of five digit numbers formed by the digits 1, 2, 3, 4, 5 is 5!.

$\therefore$ Total number of elementary events = 5! = 120.

We know that a number is divisible by 4 if the number formed by last two digits is divisible by 4.

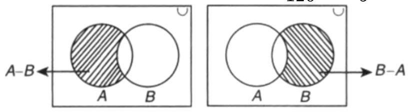
Therefore last two digits can be 12,24, 32, 52 that is, last two digits can be filled in 4 ways.

But corresponding to each of these ways there are 3! = 6 ways of filling the remaining three places.

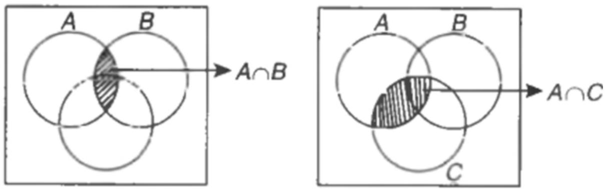
Therefore the total number of five digit numbers formed by the digits 1, 2, 3, 4, 5 and divisible by 4 is 4 $\times$6 = 24

∴ Favourable number of elementary events = 24

So, required probability = $\frac{24}{120} = \frac{1}{5}$

24.


∴ (A - B) ∪ (B - A)



∴ (A ∩ B) ∪ (A ∩ C)

or A ∩ (B ∪ C)

25. Let $\frac{x}{a} + \frac{y}{b} = 1$ be the equation of line.

It is given that a + b = 1 and ab = -6

We know that $(a - b)^2 = (a + b)^2 - 4ab$

$\Rightarrow (a-b)^2 = (1)^2 - 4 \times -6 = 1 + 24 = 25 \Rightarrow a - b = \pm 5$

Solving a + b = 1 and a - b = 5 we have

a = 3 and b = -2

Solving a +b = 1 and a - b = -5, we have

a = -2 and b = 3

Thus the required equations are

$\frac{x}{3} + \frac{y}{-2} = 1 \Rightarrow -2x + 3y = -6 \Rightarrow 2x - 3y = 6$

and $\frac{x}{-2} + \frac{y}{3} = 1 \Rightarrow 3x - 2y = -6 \Rightarrow -3x + 2y = 6$

Section C

26. To find: number of arrangements in which women sit in even places

Condition: women occupy even places

Here the total number of people is 8.

—	**W**	**—**	**W**	**—**	**W**	**—**	**W**
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**

In this question first, the arrangement of women is required.

The positions where women can be made to sit is 2^{nd}, 4^{th}, 6^{th}, 8^{th}. There are 4 even places in which 3 women are to be arranged.

Women can be placed in P (4,3) ways. The rest 5 men can be arranged in 5! ways.

Therefore, the total number of arrangements is P (4,3)×5!

Formula:

Number of permutations of n distinct objects among r different places, where repetition is not allowed, is

P(n,r) = $\frac{n!}{(n-r)!}$

Therefore, a permutation of 4 different objects in 3 places and the arrangement of 5 men are

P(4,3)×5! = $\frac{4!}{(4-3)!}$×5!

= $\frac{24}{1}$ × 120= 2880

Hence number of ways in which they can be seated is 2880

27. Let A(0, 7, 10), B(-1, 6, 6) and C(-4, 9, 6) be three vertices of triangle ABC. Then

$AB = \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2} = \sqrt{1+1+16} = \sqrt{18}$

$BC = \sqrt{(-4+1)^2 + (9-6)^2 + (6-6)^2} = \sqrt{9+9+0} = \sqrt{18}$

$AC = \sqrt{(-4-0)^2 + (9-7)^2 + (6-10)^2} = \sqrt{16+4+16} = \sqrt{36}$

$Now, (AB)^2 = 18, (BC)^2 = 18, (AC)^2 = 36$

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

Hence, ΔABC is a right-angled triangle.

28. Here $(3 + ax)^9 = {}^9C_0(3)^9 + {}^9C_1(3)^8(ax) + {}^9C_2(3)^7(ax)^2 + {}^9C_3(3)^6(ax)^3 + \dots$

$= {}^9C_0(3)^9 + {}^9C_1(3)^8 \cdot a \cdot x + {}^9C_2(3)^7(a)^2 \cdot x^2 + {}^9C_3(3)^6 \cdot a^3x^3 + \dots$

$\therefore$ Coefficient of $x^2 = {}^9C_2(3)^7a^2$

Coefficient of $x^3 = {}^9C_3(3)^6a^3$

It is given that

${}^9C_2(3)^7a^2 = {}^9C_3(3)^6a^3 \Rightarrow 36 \cdot 3^7a^2 = 84 \cdot 3^6 \cdot a^3$

$\Rightarrow a = \frac{36 \cdot 3^7}{84 \cdot 3^6} = \frac{108}{84} = \frac{9}{7}$.

OR

Let y = x + x^2. Then,

$(1 + x + x^2)^3 = (1 + y)^3 = {}^3C_0 + {}^3C_1\,y + {}^3C_2\,y^2 + {}^3C_3\,y^3 = 1 + 3y + 3y^2 + y^3$

$= 1 + 3(x + x^2) + 3(x + x^2)^2 + (x + x^2)^3$

$= 1 + 3(x + x^2) + 3(x^2 + 2x^3 + x^4) + \{{}^3C_0\,x^3\,(x^2)^0 + {}^3C_1\,x^{3-1}\,(x^2)^1 + {}^3C_2\,x^{3-2}\,(x^2)^2 + {}^3C_3\,x^0\,(x^2)^3\}$

$= 1 + 3(x + x^2) + 3(x^2 + 2x^3 + x^4) + (x^3 + 3x^4 + 3x^5 + x^6)$

$= x^6 + 3x^5 + 6x^4 + 7x^3 + 6x^2 + 3x + 1$

29. To evaluate: $\lim\limits_{x \to 2}\left(\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}\right)$

Formula used:

L'Hospital's rule

Let f(x) and g(x) be two functions which are differentiable on an open interval I except at a point a where

$\lim\limits_{x \to a} f(x) = \lim\limits_{x \to a} g(x) = 0$ or $\pm\infty$ then

$\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \lim\limits_{x \to a} \frac{f'(x)}{g'(x)}$

As x $\rightarrow$ 0, we have

$\lim\limits_{x \to 2}\left(\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}\right) = \frac{0}{0}$

This represents an indeterminate form. Thus applying L'Hospital's rule, we get

$\lim\limits_{x \to 2}\left(\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}\right) = \lim\limits_{x \to 2} \frac{\frac{d}{dx}(x^2-4)}{\frac{d}{dx}(\sqrt{x+2}-\sqrt{3x-2})}$

$\lim\limits_{x \to 2}\left(\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}\right) = \lim\limits_{x \to 2} \frac{2x}{\frac{1}{2\sqrt{x+2}} - \frac{3}{2\sqrt{3x-2}}}$

$\lim\limits_{x \to 2}\left(\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}\right) = \frac{4}{\frac{1}{2\sqrt{2+2}} - \frac{3}{2\sqrt{6-2}}}$

$\lim\limits_{x \to 2}\left(\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}\right) = \frac{8}{\frac{1}{2} - \frac{3}{2}}$

$\lim\limits_{x \to 2}\left(\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}\right)$ = -8

Thus, the value of $\lim\limits_{x \to 2}\left(\frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}\right)$ is -8

OR

Let y= $\lim\limits_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}\left[\frac{0}{0}\text{from}\right]$

Put $x = \pi + y$, as $x \to \pi,\ y \to 0$

$\therefore y = \lim\limits_{y \to 0} \frac{\sin[\pi - \pi - y]}{\pi[\pi - \pi - y]} = \lim\limits_{y \to 0} \frac{\sin(-y)}{-\pi y}$

$= \lim\limits_{y \to 0} \frac{-\sin y}{-\pi y} = \frac{1}{\pi} \lim\limits_{y \to 0} \frac{\sin y}{y} = \frac{1}{\pi} \times 1 = \frac{1}{\pi}$

30. Let A and d be the first term and common difference respectively of an A.P. and x and R be the first term and common ratio respectively of the G.P.

$\therefore$ A + (p – 1)d = a ….(i)

A + (q – 1)d = b ….(ii)

And A + (r – 1)d = c …..(iii)

For G.P., we have

xR^{p-1} = a ….(iv)

xR^{q-1} = b …..(v)

and xR^{r-1} = c …..(vi)

Subracting eq. (ii) from eq. (i) we get

(p – q)d = a – b ….(vii)

Similarly, (q – r)d = b – c ….(viii)

and (r – p)d = c – a ….(ix)

Now we have to prove that

$a^{b - c} \cdot b^{c - a} \cdot c^{a - b} = 1$

L.H.S. $a^{b - c} \cdot b^{c - a} \cdot c^{a - b}$

$= \left[xR^{p-1}\right]^{(q-r)d} \cdot \left[xR^{q-1}\right]^{(r-p)d}\left[xR^{r-1}\right]^{(p-q)}d$ [from (i), (ii), (iii), (iv), (v), (vi), (vii), (viii), (ix), (x)

$= x^{(q-r)d} \cdot R^{(p-1)(q-r)d} \cdot x^{(r-p)d} \cdot R^{(q-1)(r-p)d} \cdot x^{(p-q)d} \cdot R^{(r-1)(p-q)d}$

$= x^{(q-r)d+(r-p)d+(p-q)d} R^{(p-1)(q-r)d+(q-1)(r-p)d+(r-1)(p-q)d}$

$= x^{(q-r+r-p+p-q)d} \cdot R^{(pd-p-q+r+qr-p-r+p+pr-qr-p+q)d}$

$= x^{(0)d} \cdot R^{(0)d} = x^0 \cdot R^0$ R.H.S.

L.H.S. = R.H.S. Hence proved.

OR

Let us take a G.P. whose first is a and common difference is r.

$\therefore S_\infty = \frac{a}{1-r}$

$\Rightarrow \frac{a}{1-r} = 3$ ...(i)

And, sum of the terms of the G.P. $a^2, (ar)^2, (ar^2)^2, \ldots \infty$

$S_\infty = \frac{a^2}{1-r^2}$

$\Rightarrow \frac{a^2}{1-r^2} = \frac{9}{2}$ ...(ii)

$\Rightarrow 2a^2 = 9(1 - r^2)$

$\Rightarrow 2[3(1 - r)]^2 = 9 - 9r^2$ [From (i)]

$\Rightarrow 18(1 + r^2 - 2r) = 9 - 9r^2$

$\Rightarrow 18 - 9 + 18r^2 + 9r^2 - 36r = 0$

$\Rightarrow 27r^2 - 36r + 9 = 0$

$\Rightarrow 3(9r^2 - 12r + 3) = 0$

$\Rightarrow 9r^2 - 12r + 3 = 0$

$\Rightarrow 9r^2 - 9r - 3r + 3 = 0$

$\Rightarrow 9r(r - 1) - 3(r - 1) = 0$

$\Rightarrow (9r - 3)(r - 1) = 0$

$\Rightarrow r = \frac{1}{3}$ and r = 1

But, r = 1 is not possible.

$\therefore r = \frac{1}{3}$

Now, substituting $r = \frac{1}{3}$ in $\frac{a}{1-r}$ = 3

a = $3\left(1 - \frac{1}{3}\right)$

$\Rightarrow a = 3 \times \frac{2}{3}$ = 2

Therefore the first term is 2 and common difference is $\frac{1}{3}$

31. i. (A ∩ B) ∪ (A - B) = A

L.H.S. = (A ∩ B) ∪ (A - B)

= (A ∩ B) ∪ $(A \cap B')$ [∴ (A - B) = A ∩ B']

= A ∩ $(B \cup B')$ [By distributive law]

= A ∩ U [$(B \cup B')$= U = Universal set]

= A

= R.H.S.

ii. A ∪ (B - A) = A ∪ B

L.H.S. = A ∪ (B - A)

= A ∪ (B ∩ A') [∴ (B - A) = B ∩ A']

= (A ∪ B) ∩ $(A \cap A')$ [By distributive law]

= (A ∪ B) ∩ u [∴ A ∪ A' = u = Universal set]

= A ∪ B

= R.H.S.

Section D

32. Mean $= 7$

$\therefore \frac{\sum x_i}{18}$ = 7 [$\because$ n = 18]

$\Rightarrow \sum x_i = 18 \times 7 = 126$

Since, an observation 12 was miscopied as 21.

$\therefore$ Correct $\sum x_i = 126 - 21 + 12 = 117$

Hence, true mean = $\frac{\text{Correct } \sum x_i}{18}$ = $\frac{117}{18} = 6.5$

Also, given variance $= 4^2 = 16$

$\therefore \frac{\sum x_i^2}{18}$ - (Mean)$^2 = 16$

$\Rightarrow \frac{\sum x_i^2}{18}$ = 16 + (Mean)$^2 = 16 + (7)^2$

$\Rightarrow \frac{\sum x_i^2}{18} = 16 + 49$

$\Rightarrow \sum x_i^2 = 18 \times 65 = 1170$

But one observation 12 was miscopied as 21.

Correct $\sum x_i^2 = 1170 - 21^2 + 12^2 = 1170 - 441 + 144 = 873$

Hence, correct variance = $\frac{\text{Correct } \sum x_i^2}{18} - (\text{ Correct mean })^2$

= $\frac{873}{18} - (6.5)^2 = 48.5 - 42.25 = 6.25$

$\therefore$ Correct standard deviation = $\sqrt{Correct\ variance}$

= $\sqrt{6.25} = 2.5$

33. Given: $16x^2 + 25y^2 = 400$

After dividing by 400 to both the sides, we get

$\frac{16}{400}x^2 + \frac{25}{400}y^2$ = 1

$\frac{x^2}{25} + \frac{y^2}{16}$ = 1 ...(i)

Now, above equation is of the form,

$\frac{x^2}{a^2} + \frac{y^2}{b^2}$ = 1 ...(ii)

Comparing eq. (i) and (ii), we get

$a^2 = 25$ and $b^2 = 16 \Rightarrow a = 5$ and b = 4

i. Length of major axes

   $\therefore$ Length of major axes = 2a = 2 $\times$ 5 = 10 units

ii. Coordinates of the Vertices

   $\therefore$ Coordinate of vertices = (a, 0) and (-a, 0) = (5, 0) and (-5, 0)

iii. Coordinates of the foci

   As we know that

   Coordinates of foci = ($\pm$c, 0)

   Now $c^2 = a^2 - b^2 = 25 - 16$

   $\Rightarrow c^2 = 9 \Rightarrow c = \sqrt{9}$ $\Rightarrow$ c = 3...(iii)

   $\therefore$ Coordinates of foci = $(\pm 3, 0)$

iv. Eccentricity

   As we know that, Eccentricity = $\frac{c}{a} \Rightarrow e = \frac{3}{5}$ [from (iii)]

v. Length of the Latus Rectum

   As we know, Length of Latus Rectum = $\frac{2b^2}{a} = \frac{2\times(4)^2}{5} = \frac{32}{5}$

OR

We have,

$x^2 - 2y^2 - 2x + 8y - 1 = 0$

$\Rightarrow (x^2 - 2x) - 2(y^2 - 4y) = 1$

$\Rightarrow (x^2 - 2x + 1) - 2(y^2 - 4y + 4) = -6$

$\Rightarrow (1 - x)^2 - 2(y - 2)^2 = -6$

$\Rightarrow \frac{(x-1)^2}{(\sqrt{6})^2} - \frac{(y-2)^2}{(\sqrt{3})} = -1$ ... (i)

Shifting the origin at (1, 2) without rotating the coordinate axes and denoting the new coordinates with respect to these axes by X and Y, we obtain

x = X + 1 and y = Y + 2 ... (ii)

Using these relations, equation (i) reduces to

$\frac{X^2}{(\sqrt{6})^2} - \frac{Y^2}{(\sqrt{3})^2} = -1$ ... (iii)

Comparing equation (iii) with standard form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$, we get

$a^2 = (\sqrt{6})^2$ and $b^2 = (\sqrt{3})^2$

$a = (\sqrt{6})$ and $b = (\sqrt{3})$

Centre:

The coordinates of the centre with respect to the new axes are (X = 0, Y = 0).

So, the coordinates of the centre with respect to the old axes are

(1, 2) [Putting X = 0, Y = 0 in (ii)]

Lengths of the axes:

Since the transverse axis of the hyperbola is along new T-axis.

$\therefore$ Transverse axis = 2b = 2 $\sqrt{3}$ and, Conjugate axis = 2a = 2 $\sqrt{6}$.

Eccentricity:

$e = \sqrt{1 + \frac{a^2}{b^2}} = \sqrt{1 + \frac{6}{3}} = \sqrt{3}$

Latusrectum:

Length of the latus rectum $= \frac{2a^2}{b} = \frac{12}{\sqrt{3}} = 4\sqrt{3}$

Foci:

The coordinates of foci with respect to the new axes are (X = 0, Y = $\pm$ be) i.e. (X = 0, y = $\pm$ 3). So, the coordinates of foci with respect to the old axes are

(1, 2 $\pm$ 3) i.e. (1, 5) and (1, -1) [Putting X = 0 ,y = $\pm$ 3 in (ii)]

Vertices:

The coordinates of the vertices with respect to the new axes are X = 0, Y = $\pm$ b) i.e. (x = o ,y = $\pm\sqrt{3}$)

So, the coordinates of the vertices with respect to the old axes are

(1, 2 $\pm\sqrt{3}$) i.e. (1, 2 + $\sqrt{3}$) and (1, 2 - $\sqrt{3}$) [Putting X = 0, Y = $\pm\sqrt{3}$ in (ii)]

Directrices:

The equations of the directrices with respect to the new axes are Y = $\pm$ $\frac{b}{e}$ i.e. y = $\pm$ 1.

So, the equations of the directrices with respect to the old axes are

y = 2 $\pm$ 1 i.e. y = 1 and y = 3 [Putting Y = $\pm$ 2 in (ii)]

34. We have, $\frac{|x+3|+x}{x+2} > 1$

$\Rightarrow \frac{|x+3|+x}{x+2} - 1 > 0$

$\Rightarrow \frac{|x+3|+x-x-2}{x+2} > 0$

$\Rightarrow \frac{|x+3|-2}{x+2} > 0$

Let x + 3 = 0

$\Rightarrow$ x = - 3

$\therefore$ x = - 3 is a critical point.

So, here we have two intervals $(-\infty, -3)$ and $[-3, \infty)$

Case I: When - 3 $\leq$ x < ∞, then $|x+3|$= (x + 3)

$\therefore \frac{|x+3|-2}{x+2} > 0$

$\Rightarrow \frac{x+3-2}{x+2} > 0$

$\Rightarrow \frac{x+1}{x+2} > 0$

$\Rightarrow \frac{(x+1)(x+2)^2}{(x+2)} > 0 \times (x+2)^2$

$\Rightarrow$ (x + 1) (x + 2) > 0

Product of (x + 1) and (x + 2) will be positive, if both are of same sign.

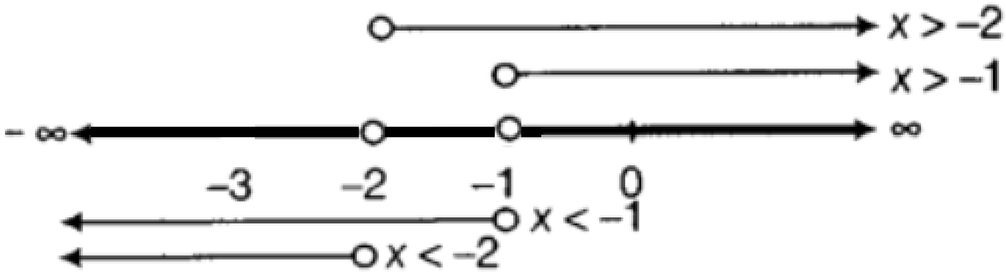
$\therefore$ (x + 1) > 0 and (x + 2) > 0

or (x + 1) < 0 and (x + 2) < 0

$\Rightarrow$ x > - 1 and x > - 2

or x < - 1 and x < - 2

On number line, these inequalities can be represented as,



Thus, - 1 < x < ∞ or - ∞ < x < - 2

But, here - 3 $\leq$ x < ∞

$\therefore$ - 1 < x < ∞ or - 3 $\leq$ x < - 2

Then, solution set in this case is

$x \in$ [- 3, - 2) $\cup$ (- 1, ∞)

Case II: When x < - 3, then $|x+3|$ = - (x + 3)

$\therefore \frac{|x+3|-2}{x+2} > 0$

$\Rightarrow \frac{-x-3-2}{x+2} > 0$

$\Rightarrow \frac{-(x+5)}{x+2} > 0$

$\Rightarrow \frac{x+5}{x+2} < 0$

$\Rightarrow \frac{(x+5)(x+2)^2}{x+2} < 0 \times (x+2)^2$

$\Rightarrow$ (x + 5) (x + 2) < 0

Product of (x + 5) and (x + 2) will be negative, if both are of opposite sign.

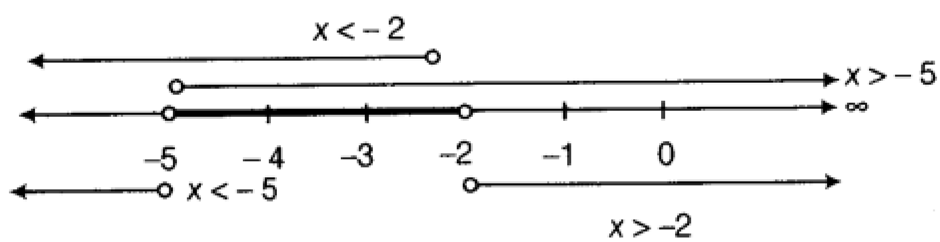
$\therefore$ (x + 5) > 0 and (x + 2) < 0

or (x + 5) < 0 and (x + 2) > 0

$\Rightarrow$ x > - 5 and x < - 2

or x < - 5 and x > - 2

On number line, these inequalities can be represented as,



Thus, - 5 < x < - 2 i.e., solution set in the case is $x \in$ (- 5, - 2).

On combining cases I and II, we get the required solution set of given inequality, which is

$x \in$ (- 5, - 2) $\cup$ (- 1, ∞)

35. **LHS** $= cos12^o + cos60^o + cos84^o$

$= cos12^o + (cos84^o + cos60^o)$

$= cos12^o + [2cos\left(\frac{84^\circ+60^\circ}{2}\right) \times cos\left(\frac{84^\circ-60^\circ}{2}\right)]$

[$\because$ cosx +cosy = 2 cos $\left(\frac{x+y}{2}\right)$ cos $\left(\frac{x-y}{2}\right)$]

$= cos12^o + [2cos\ \frac{144^\circ}{2} \times \cos\ \frac{24^\circ}{2}]$

$= cos12^o + [2cos72^o \times cos12^o] = cos12^o[1 + 2cos72^o]$

$= cos12^o[1 + 2cos(90^o - 18^o)]$

= cos 12^o [1 + 2 sin 18^o] [$\because$ cos (90^o - θ) = sin θ]

= cos 12^o [1 + 2 $\left(\frac{\sqrt{5}-1}{4}\right)$] [$\because$ sin 18^o = $\frac{\sqrt{5}-1}{4}$]

= $\left(1 + \frac{\sqrt{5}-1}{2}\right)$ cos 12^o = $\left(\frac{\sqrt{5}+1}{2}\right)$ cos 12^o

RHS = cos 24^o + cos 48^o

= 2 cos $\left(\frac{24^\circ+48^\circ}{2}\right)$ cos $\left(\frac{24^\circ-48^\circ}{2}\right)$ [$\because$ cosx + cosy = 2 cos $\left(\frac{x+y}{2}\right)$ cos $\left(\frac{x-y}{2}\right)$]

$= 2cos36^o cos(-12^o)$

= 2 cos $36^o \times$ cos 12^o [$\because$ cos (- θ) = cos θ]

= 2 $\times \frac{\sqrt{5}+1}{4} \times$ cos 12^o = $\frac{\sqrt{5}+1}{2} \times$ cos 12^o [$\because$ cos $36^o = \frac{\sqrt{5}+1}{4}$]

$\therefore$ **LHS = RHS**

Hence proved.

OR

We have to prove that sin 5x = 5 sin x - 20 $\sin^3$ x + 16 $\sin^5$ x.

Let us consider LHS = sin 5x

sin 5x = sin(3x + 2x)

But we know,

sin(x + y) = sin x cos y + cos x sin y ... (i)

⇒ sin 5x = sin 3x cos 2x + cos 3x sin 2x

⇒ sin 5x = sin (2x + x) cos 2x + cos (2x + x) sin 2x ... (ii)

And

cos (x + y) = cos(x)cos(y) - sin(x)sin(y) ... (iii)

Now substituting equation (i) and (iii) in equation (ii), we get

$\Rightarrow$ sin 5x = (sin 2x cos x + cos 2x sin x)cos 2x + (cos 2x cos x - sin 2x sin x) sin 2x

$\Rightarrow$ sin 5x = sin 2x cos 2x cos x + $\cos^2$ 2x sin x + (sin 2x cos 2x cos x - $\sin^2$ 2x sin x)

$\Rightarrow$ sin 5x = 2sin 2x cos 2x cos x + $\cos^2$ 2x sin x - $\sin^2$ 2x sin x ... (iv)

Now sin 2x = 2sin x cos x ... (v)

And cos 2x = $\cos^2$x - $\sin^2$x ... (vi)

Substituting equation (v) and (vi) in equation (iv), we get

$\Rightarrow$ sin 5x = 2(2sin x cos x)($\cos^2$x - $\sin^2$x)cos x + ($\cos^2$x - $\sin^2$x$)^2$sin x - (2sin x cos x$)^2$sin x

$\Rightarrow$ sin 5x = 4(sin x $\cos^2$ x)([1 - $\sin^2$x] - $\sin^2$x) + ([1 - $\sin^2$x] - $\sin^2$x$)^2$sin x - (4$\sin^2$ x $\cos^2$ x)sin x (as $\cos^2$x + $\sin^2$x = 1 $\Rightarrow$ $\cos^2$x = 1 - $\sin^2$x)

$\Rightarrow$ sin 5x = 4(sin x [1 - $\sin^2$x])(1 - 2$\sin^2$x) + (1 - 2$\sin^2$x$)^2$sin x - 4$\sin^3$ x [1 - $\sin^2$x]

$\Rightarrow$ sin 5x = 4sin x(1 - $\sin^2$x)(1 - 2$\sin^2$x) + (1 - 4$\sin^2$x + 4$\sin^4$x)sin x - 4$\sin^3$ x + 4$\sin^5$x

$\Rightarrow$ sin 5x = (4sin x - 4$\sin^3$x)(1 - 2$\sin^2$x) + sin x - 4$\sin^3$x + 4$\sin^5$x - 4$\sin^3$ x + 4$\sin^5$x

$\Rightarrow$ sin 5x = 4sin x - 8$\sin^3$x - 4$\sin^3$x + 8$\sin^5$x + sin x - 8$\sin^3$x + 8$\sin^5$x

$\Rightarrow$ sin 5x = 5sin x - 20$\sin^3$x + 16$\sin^5$x

Hence LHS = RHS

Hence proved.

Section E

36. i. Number of relations = 2^{mn}

   = $2^{3\times6} = 2^{18}$

 ii. Number of relations = 2^{mn}

   = $2^{2\times2} = 2^4$ = 16

 iii. R = {(x, y): x $\in$ P, y $\in$ Q and x is the square of y}

   OR

   Here, W denotes the set of whole numbers.

   We have 2a + b = 5 where a, b $\in$ W

   $\therefore$ a = 0 $\Rightarrow$ b = 5

   $\Rightarrow$ a = 1 $\Rightarrow$ b = 5 − 2 = 3

   and a = 2 $\Rightarrow$ b = 1

   For a > 3, the values of b given by the above relation are not whole numbers.

   $\therefore$ A = {(0, 5), (1, 3), (2, 1)}

37. i. Total marbles = 4 + 5 + 3 = 12

   Required probability $= \frac{^4C_2}{^{12}C_2} = \frac{\frac{4\times3}{2\times1}}{\frac{12\times11}{2\times1}} = \frac{1}{11}$

ii. Total marbles = 4 + 5 + 3 = 12

Required probability = $\frac{^3C_3}{^{12}C_3} = \frac{1}{\frac{12\times11\times10}{3\times2}} = \frac{1}{220}$

iii. Total marbles = 4 + 5 + 3 = 12

Required probability $= \frac{^7C_2}{^{12}C_2} = \frac{\frac{7\times6}{2\times1}}{\frac{12\times11}{2\times1}} = \frac{21}{66} = \frac{7}{22}$

OR

Total marbles = 4 + 5 + 3 = 12

Required probability = 1 - P (None is blue)

$= 1 - \frac{^7C_3}{^{12}C_3}$

$= 1 - \frac{\frac{7\times6\times5}{3\times2}}{\frac{12\times11\times10}{3\times2}}$

$= 1 - \frac{7}{44} = \frac{37}{44}$

38. i. $Z_1\overline{Z_1}$ = (2 + 3i)(2 - 3i)

= 4 - 9i^2 = 4 + 9 = 13

Imaginary part = 0

ii. $\frac{Z_1}{Z_2} = \frac{2+3i}{4-3i} \times \frac{4+3i}{4+3i}$

= $\frac{8+6i+12i-9}{16+9}$

= $\frac{-1+18i}{25}$

Real part = $\frac{-1}{25}$

iii. $Z_1 - Z_2$ = (2 + 3i) - (4 - 3i)

= -2 + 6i

Imaginary part = 6

OR

The real part of Z_1 = 2.